Spring 2010

Physics 137B: Problem Set #11
Due: 5PM Friday April 30 in the appropriate dropbox inside 251 LeConte (the "reading room")

Suggested Reading for this Week:

- B&J 13.7 and 13.8
- B&J 16.3

Homework Problems:

1. Particle A hits particle B and scatters. The scattering amplitude for two particles is given by the form

$$f(\theta) = a + b\cos\theta$$

   Find the total cross section for particle A if

   (a) A and B are distinguishable particles

   (b) A and B are identical spin-0 particles

   (c) A and B are identical spin-$\frac{1}{2}$ particles and both are initially unpolarized

   (d) A and B are identical spin-$\frac{1}{2}$ particles and both have $s_z = \frac{1}{2}\hbar$ before they scatter

2. (Taken from D.H. Lee's class in Fall 2009) This problem concerns the principle of neutron scattering. Consider a neutron (spin 1/2) incident on a magnetic material. Let us suppose this magnetic material can be thought of as a collection of spin 1/2 particles localized on a three dimensional lattice r $= (n_x; n_y; n_z)a$. In the absence of coupling the Hamiltonian of the problem is:

$$H_0 = -\frac{\hbar^2}{2m_N}\sum_{i,j} J_{ij}\vec{S}_i \cdot \vec{S}_j$$

Here i, j labels the lattice sites of the magnetic solid. The eigenstates of H_0 are the direct product of those of neutron and those of the magnetic solid, i.e.,

$$\left|\vec{k}, \sigma, \alpha\right\rangle = \left|\vec{k}, \sigma\right\rangle |\alpha\rangle$$

The eigen-energy of H0 is

$$E_{k,\sigma\alpha} = \frac{\hbar^2 k^2}{2m_N} + E_\alpha$$

Here $\vec{k}$ is the wave vector of the neutron, $\sigma = +, -$ labels the neutron spin, and α labels the eigenstate of the magnetic solid. (Writing down the explicit expressions for E_α and $|\alpha\rangle$ is a formidable task, but for this homework problem you don't have to know the explicit form.) The interaction between the neutron and the magnetic spins:

$$V(r) = 2K \sum_j \delta(\vec{r} - \vec{r_j}) \vec{S} \cdot \vec{S_j}$$

Here $\vec{r}$ is the coordinate of the neutron, $\vec{S}$ is the spin operator of the neutron, and $\vec{r_j}$ is a lattice position. Now suppose initially the neutron is in the plane wave state $\vec{k_i}$ with spin up, and the magnetic solid is in its ground state $|0\rangle$ (E_0 is the ground state energy

(a) Write down the expression for spin-conserving elastic scattering differential cross section. (The neutron scatters from the initial state $\vec{k_i}$; to a final state where the spin is unchanged but the momentum falls within solid angle $d\Omega$ around direction $\hat{n}$.)

(b) Write down the expression for spin flipping inelastic scattering differential cross section. (The neutron scatters from the initial state $\vec{k_i}$; to a final state where the spin is reversed but the momentum falls within solid angle $d\Omega$ around direction $\hat{n}$, and the magnetic solid is excited from it ground state to first excited state.)